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Differences in 1D electron plasma wake field acceleration in MeV versus GeV and linear versus blowout regimes

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In some laboratory and most astrophysical situations, plasma wake-field acceleration of electrons is one dimensional, i.e., variation transverse to the beam’s motion can be ignored. Thus, one dimensional, particle-in-cell (PIC), fully electromagnetic simulations of electron plasma wake field acceleration are conducted in order to study the differences in electron plasma wake field acceleration in MeV versus GeV and linear versus blowout regimes. First, we show that caution needs to be taken when using fluid simulations, as PIC simulations prove that an approximation for an electron bunch not to evolve in time for a few hundred plasma periods only applies when it is sufficiently relativistic. This conclusion is true irrespective of the plasma temperature. We find that in the linear regime and GeV energies, the accelerating electric field generated by the plasma wake is similar to the linear and MeV regimes. However, because GeV energy driving bunch stays intact for a much longer time, the final acceleration energies are much larger in the GeV energies case. In the GeV energy range and blowout regime, the wake’s accelerating electric field is much larger in amplitude compared with the linear case and also plasma wake geometrical size is much larger. Thus, the correct positioning of the trailing bunch is needed to achieve the efficient acceleration. For the considered case, optimally, there should be approximately (90–100)c/ωpe distance between the trailing and driving electron bunches in the GeV blowout regime. Published by AIP Publishing.

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I. INTRODUCTION

The plasma acceleration based on laser field acceleration stems from a paper by Tajima and Dawson.1 When a laser is injected in plasma, a ponderomotive force arises from the nonlinear Lorentz force v × B/c, which causes the polarization of electrons in the plasma in the longitudinal direction, even though the electric field of the laser is in the transverse direction. This polarization E_p = meν/ca0e yields the electrostatic field in the longitudinal direction with the same order of magnitude. Here, the normalized vector potential of the laser is a0 = eE0m_e/c0 and E0, ω0 are the electric field and frequency of the laser, respectively. The relativistically intense laser makes the amplitude of the wake-field E_p relativistically intense, i.e., a_p = eE_pm_eω_p/c ≥ 1.2 The plasma wave breaking occurs at a0 ≈ 1. The experimental implementation of the plasma wake field was done by Joshi.3 There are two main possibilities for the creation of plasma wake: a laser or an electron bunch. The former is referred to as laser wake field acceleration (LWFA) and the latter as plasma wake field acceleration (PWFA). In early experiments, injected electrons of a few MeV have been accelerated by GV/m electric fields using LWFA.4 In these experiments, because the bunch length of the injected electrons was much longer than the plasma wavelength, only a small fraction of the injected electrons were accelerated. In turn, this results in a poor electron beam quality.5 More recent experimental devices with compact electron beams show accelerating gradients that are several orders of magnitude better (10 s of GeV m⁻¹) than the current RF-based conventional particle accelerators (10 s of MeV m⁻¹). A significant progress in PWFA has been made recently in both experiment and theory.6–10

Good progress has been made in applying PWFA concepts to astrophysical plasmas. This includes supermassive black hole11,12 and solar atmosphere13 contexts.

Section II presents the model and results. In Sec. III, we list the main findings.

II. THE MODEL AND RESULTS

We used EPOCH, a fully electromagnetic (EM), relativistic particle-in-cell (PIC) code14 for the simulation. EPOCH is available for download from https://cfsa.pmmw.warwick.ac.uk. The mass ratio in all runs is m_p/m_e = 1836.153, and the boundary conditions are periodic. The choice of boundary conditions is not important here because the simulation domain is long enough, such that the electron bunches never reach the boundary.

The simulation domain is split into n_x = 65 280 grid cells in the x-direction. We fix the grid size Δ as Debye length (λ_D) times appropriate factor (f), i.e., Δ = fλ_D. Here, λ_D = v_th,e/ω_pe denotes the Debye length with v_th,e = √k_BT/me being the electron thermal speed and ω_pe the electron plasma frequency. In the plasma wake field acceleration, the relevant spatial scale is electron inertial length, c/ω_pe. We vary the factor f such that: (i) in runs 1–3, f = 1 and c/ω_pe is resolved with 243 grid points, i.e., (c/ω_pe)/Δ = 243.5, where again Δ = fλ_D is the grid size; (ii) in runs 4 and 5, f = 10 and c/ω_pe is also resolved with 24 grid points, i.e., (c/ω_pe)/Δ = 24.35. This choice provides a good resolution as the energy error never exceeds ≈0.0004%.
Different numerical runs are described in Table I. At $t = 0$, the driving and trailing electron bunches have the number densities as follows:

$$n_D(x) = A_Dn_0 \exp\left[-\frac{(x - x_Dc/\alpha_{pe})^{16}}{(4.0c/\alpha_{pe})^{16}}\right],$$  \hspace{1cm} (1)

$$n_T(x) = A_Tn_0 \exp\left[-\frac{(x - \frac{\gamma}{\gamma-1}c\alpha_{pe})^{16}}{(c/\alpha_{pe})^{16}}\right],$$  \hspace{1cm} (2)

where $x_D$ is the location of the driving bunch in units of $c/\alpha_{pe}$ and it has a length of $4.0c/\alpha_{pe}$, as in Figs. 1 and 2 from Bera et al.,\textsuperscript{10} for easy inter-comparison. $x_D = 10c/\alpha_{pe}$ in Runs 1–4 and $x_D = 94.5c/\alpha_{pe}$ in Run 5. Trailing bunch that is absent in Ref. 10 simulation is located at $4.5c/\alpha_{pe}$ and has a length of $c/\alpha_{pe}$. $A_D$ and $A_T$ are the bunch amplitudes in units of $n_0$. If a trailing bunch is present, then both electron bunch initial momenta are set to values shown in Table I. For example, for Runs 2 and 3, we set $p_x = p_0 = \gamma m_0 c 0.9999 \text{kg m s}^{-1}$ [note that $p_x/(m_0c) = 70.7$, i.e., $\gamma = 70.7$], which corresponds to an initial energy of $E_0 = 36.1 \text{MeV}$. In the simulation with both bunches, there are four plasma species present: background electrons and ions, plus driving and trailing bunches. In the numerical runs, there are 256 particles per cell for each of the four species. The numerical Runs 1–3 take about 3 h on 96 cores using Intel Xeon E5-2683V3 (Broadwell) processors with 256 GB of RAM and Mellanox ConnectX-4 EDR Infiniband Interconnect. Runs 4 and 5 take 5 h each on 192 cores of the same processors.

### Table I. Conducted numerical run details.

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_b$</th>
<th>$E_i$</th>
<th>$t_{end}$</th>
<th>$E_{f,T}$</th>
<th>$A_D$</th>
<th>$A_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>1</td>
<td>3.6 MeV</td>
<td>200</td>
<td>N/A</td>
<td>0.3</td>
<td>N/A</td>
</tr>
<tr>
<td>Run 2</td>
<td>1</td>
<td>36 MeV</td>
<td>200</td>
<td>N/A</td>
<td>0.3</td>
<td>N/A</td>
</tr>
<tr>
<td>Run 3</td>
<td>2</td>
<td>36 MeV</td>
<td>200</td>
<td>85 MeV</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Run 4</td>
<td>2</td>
<td>20 GeV</td>
<td>2000</td>
<td>21 GeV</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Run 5</td>
<td>2</td>
<td>20 GeV</td>
<td>2000</td>
<td>24 GeV</td>
<td>2.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*FIG. 1. (a)–(c) Electric field x-component at different time instants corresponding to 1/5th, half, and the final simulations times. (d)–(f) log normal plot of driving electron bunch number density at the same times. (g)–(i) Solid line is the background electron number density, in units of $m_0^{-1}$, at the same times. Dashed line is the initial $(t = 0)$ background electron number density of $5 \times 10^{22} \text{m}^{-3}$, to guide the eye. The fields are quoted in $V/m$ and time at the top of each panel is in $x_{pe}$. The data are for Run 1. See text and Table I for details. Note that the x-coordinate is different in each raw because we use a window which follows the bunch with speed $v_b$.**
A. Runs 1 and 2—importance of large $\gamma$-factor

In Run 1, we put to test fluid-simulation results of Bera et al. The physical parameters are as in their Fig. 1 and are stated in Table I.

Figure 1 top row, (a)–(c), shows the electric field $x$-component at different time instants corresponding to 1/5th, half, and the final simulations times. We see that as the electron bunch moves in plasma, it generates a wake, and at early times, electric field is of the order of $10^{10}$ V/m, but by the end, the simulation time is depleted to $10^{9}$ V/m. This is understandable, because, as we see in Figs. 1(d)–1(f), where we show log normal plots of driving electron bunch number density at the same times, the electron beam completely disintegrates. Commensurate plots in Figs. 1(g)–1(i) of background electron number density also show that the wake weakens. Figure 2 shows the results of Run 2 which has all the same parameters of Run 1, except $v_b = 0.9999c$ ($E_0 = 36$ MeV), while in Run 1, $v_b = 0.99c$ ($E_0 = 3.6$ MeV). We gather from Figs. 2(a)–2(c) that the electric field of the wake is not changing significantly (noticeably) in time. Also, Figs. 2(d)–2(f) show that the electron bunches stay intact and only start showing minor shape distortions in the leading bunch by time $t_{\omega_{pe}} = 200$. We see from Figs. 2(g)–2(i) that the electron number density is also stable and resembles closely both analytical solutions and fluid simulation results shown in Fig. 1 of Bera et al. We do not overplot their analytical solution for brevity here, but the closeness of match is obvious. The main conclusion from Figs. 1 and 2 particularly from respective Figs. 2(d)–2(f) is that caution is needed when using fluid simulations, as our PIC simulations prove that an approximation for an electron bunch not to evolve in time for few hundred plasma periods only applies when it is sufficiently relativistic, i.e., $v_b = 0.9999c$ and not mere $v_b = 0.99c$.

B. Run 3—acceleration in the MeV energy range

Now that we established that for $v_b = 0.9999c$, the plasma wake is stable and also judging from the structure and spatial extent of negative values of electric field from Figs. 2(a)–2(c), we perform new Run 3, now also adding a trailing bunch of length $c/\omega_{pe}$ and at a position prescribed by Eq. (2). Such length and position deliberately coincide with a large negative dip shown in Figs. 2 and 3(a)–3(c).

According to Bulanov et al., there are two effects that work against an efficient electron acceleration: (i) depletion of either driving laser pulse or electron bunch and (ii) de-phasing of the trailing electron bunch from the negative electrostatic $E_x$ plasma wake. Naturally, only negative electric field can accelerate the electrons. The positive one, on the contrary, produces deceleration. Both the electron slip-page with respect to the accelerating phase of the wake and the driving bunch/laser pulse energy depletion are important. Comparing (a)–(c) with (d)–(f) in Fig. 3, we see that the
trailing bunch is co-spatial with a negative dip in $E_x$. This means that the driving bunch will be decelerating, while the trailing bunch accelerating, because of sign of $E_x$. Note that in the end of simulation time $t_{\omega_{pe}} = 200$, in Fig. 3(f), the leading bunch starts to develop some rather localized spikes. This means that the first effect (depletion) comes into play. We thus stop simulation at $t_{\omega_{pe}} = 200$. From the Figs. 3(g)–3(i), we gather that initially there was a substantial cavity created in the background electrons.

In Fig. 4, we plot the background electron (a), ion (b), trailing (c), and driving (d) electron bunch distribution functions at different times in different colors: open diamonds correspond to $t = 0$, while blue and red curves to the half and the final simulations times, respectively. These data correspond to Run 3. We gather from Fig. 4(a) that background electrons develop broad peaks corresponding roughly to the momenta $p_x/\sqrt{m_e c^2} \approx 0.3$, which is probably due to acceleration of trapped background electrons in positive and negative peaks of $E_x$. In Fig. 4(b), we see that background ions develop a beam with positive velocities corresponding to momentum $p_x/\sqrt{m_i c^2} \approx 0.0008$. Figure 4(c) shows that by the end of simulation, the trailing bunch gains energy to 85 MeV (red curve), starting from initial 36.1 MeV. Figure 4(d) shows that by the end of simulation, the driving bunch loses energy to 0.5 MeV (red curve), starting from initial 36.1 MeV. This demonstrates that trailing electron bunch acceleration is due to deceleration of the driving bunch. The same conclusion follows from the dynamics of different kinds of energies in Fig. 5.

In Fig. 5(a), solid and dashed curves are the total (particles plus EM fields) and only particle energies, respectively, normalized on their initial values. Figure 5(b) shows the EM field energy, normalized on its final simulation time value. Because at $t = 0$, all EM fields are zero, hence the initial EM field energy cannot be used for normalization. The data are for Run 3. The total normalized energy stays constant and is approximately unity. Its maximal deviation from unity is 0.000004, i.e., 0.0004% which is due to numerical heating and numerical dissipation (due to finite differencing). The particle energy decreases by 2.5%. The particle energy decreases because of deceleration of driving bunch which then generates plasma wake (essentially relativistic Langmuir waves) which is then absorbed by the trailing bunch. Although the trailing bunch is accelerated overall, particle energy decreases as the driving bunch is 4 times longer in the x-direction. Accordingly, we see from Figure 5(b) that the EM field energy normalized to its final simulation time value increases, commensurately to the decrease in the particle energy in Figure 5(a).

C. Run 4—acceleration in the GeV energy range, linear regime

In Run 4, we keep everything as in Run 3, but we increase the energy of driving and trailing bunches to...
20.4 GeV also since the increase in energy allows for the driving bunch to move longer distances without depletion, we let it run for ten times longer time interval, i.e., to $t_{xpe} = 2000$. We also make simulation box ten times longer, while keeping the same number of grid points, i.e., $c/\omega_{pe}$ is resolved with also 24 grid points, which is a tolerable resolution.

Note that unlike in Run 3, where by time $t_{xpe} = 200$, the driving bunch started to show signs of depletion, in Run 4, by time $t_{xpe} = 2000$, the bunch stays intact [see Figs. 6(d)–6(f)] and, in principle, it would have been possible to continue the simulation and accelerate the trailing bunch to even higher energies.

We gather from Fig. 7(c) that the trailing bunch accelerated to approximately 21 GeV starting from 20.4 GeV. This is because, as can be seen in Figs. 6(a)–6(f), the trailing bunch rides in the negative wake-field of about $-10^{10}$ V/m.

D. Run 5—acceleration in the GeV energy range, blowout regime

In Run 5, we keep everything as in Run 4, but now we employ blowout regime by increasing driving and trailing bunch number densities, as stated in Table I. The motivation for considering this numerical run was to see whether trailing bunch acceleration is possible in one dimensional (1D) and blowout regime at the same time. It is well known that in the blowout regime of plasma wake-field acceleration, it is mostly the transverse electric field that creates the density cavity (sometimes also called the bubble) behind the driving electron bunch. In 1D, electrons cannot move in transverse direction; that is why, Tsiklauri concluded that they have not seen trailing bunch acceleration in their 1D simulations. As will be shown later, this conclusion is only partially correct and placing the trailing bunch in a suitable position makes its acceleration possible. Here, we explored this topic further. Based on various runs to optimize the acceleration, it was found that Run 5 provides favorable conditions.

Namely, we had to increase the distance between the driving and trailing bunches from 10–4.5 to 90.0$c/\omega_{pe}$ in Run 5. This was done in order to place the trailing bunch into middle of the negative electric field wake, as can be seen in Figs. 8(a)–8(f). We also note from Fig. 8(c) that despite the fact driving bunch stays intact [see Fig. 8(f)], the plasma wake becomes quite distorted by the end of simulation [see Fig. 8(c)]. This is quite different from Runs 1 to 4. At this stage, it is unclear what is the source of such distortion of $E_x(x, t_{xpe} = 2000)$.

Nonetheless, as can be seen from Fig. 9(c), starting from initial 20.4 GeV, the trailing bunch accelerates to
24 GeV. So, it is possible to have plasma wake-field acceleration in 1D and blowout regimes. It was found that, optimally, there should be approximately $(90–100)c/\omega_{pe}$ distance between the trailing and driving electron bunches, because in the 1D blowout regime, driving bunch’s wake is much longer than in 3D. As in the 1D case, electrons cannot move in the transverse direction, and the wake becomes much longer compared with 3D (and 2D).

FIG. 6. As in Fig. 3 but for Run 4.

FIG. 7. As in Fig. 4 but for Run 4. Note that energies in the red, top scale in (c) and (d) are now in GeV.
FIG. 8. As in Fig. 3 but for Run 5.

FIG. 9. As in Fig. 4 but for Run 5. Again, energies in the red, top scale in (c) and (d) are in GeV.
E. Investigation of plasma temperature dependence

In all numerical Runs 1–5, the temperature of all plasma species with set equal to $T = 10^5$ K. Fluid-simulation results of Bera et al.\textsuperscript{10} were carried out for cold beam-plasma system, while our numerical simulations have finite temperature. In this subsection, we aim to investigate whether our conclusion that an approximation for an electron bunch not to evolve in time for few hundred plasma periods only applies when it is sufficiently relativistic, i.e., $v_b = 0.9999c$ and not mere $v_b = 0.99c$, depends on temperature variation. In Fig. 10, we present additional simulation results as in Fig. 1, but for numerical simulation similar to Run 2, now with 100 times hotter temperature and 10 times shorter domain length. Because our grid length, as in every other PIC simulation, is proportional to the Debye length, which in turn is proportional to $v_{th,e} = \sqrt{k_BT/m_e}$ (i.e., $\lambda_D = v_{th,e}/\omega_{pe}$), 100 times hotter plasma requires 10 shorter domain length otherwise end simulation time of $200/\omega_{pe}$ would have to be altered. For clear comparison, we wanted to keep the same end simulation time. We gather from Fig. 10 that results are not significantly different from Fig. 2.

In Fig. 11, we present additional simulation results as in Fig. 1 but for numerical simulation similar to Run 2, now with 100 times cooler temperature and 10 times longer domain length. Again, we see in Fig. 11 that the results are not significantly different from Fig. 2. Thus, from both Figs. 10 and 11, our conclusion is not temperature dependent. The fact that in fluid simulation of Bera et al.\textsuperscript{10} the beam is intact even for $v_b = 0.99c$ and can be explained by the fact that they (fluid-approach) ignore wave–particle interactions. Thus, in finite-temperature PIC simulation, it is rather important to have electron beam with speed very close to the speed of light.

III. CONCLUSIONS

Here, we argue that in some laboratory, see, e.g., Fig. 1 from Corde et al.\textsuperscript{16} and probably most astrophysical situations such as jets in the vicinity of black holes\textsuperscript{11,12} and flares in solar atmosphere,\textsuperscript{13} plasma wake-field acceleration of electrons is one dimensional. Namely, variation transverse to the beam’s motion can be ignored. Thus, one dimensional (1D), particle-in-cell (PIC), fully electromagnetic simulations of electron plasma wake field acceleration were conducted in order to study the differences in electron plasma wake field acceleration in MeV versus GeV and linear versus blowout regimes. First, it has been shown that case needs to be taken when using fluid simulations, as PIC simulations demonstrate that an approximation for an electron bunch not to evolve in time for few hundred plasma periods only applies when it is sufficiently relativistic. The electron bunch speed needs to be at least $v_b = 0.9999c$ and not just $v_b = 0.99c$. We establish that injecting driving and trailing electron bunches into plasmas with $n_0 = 5 \times 10^{22}$ m$^{-3}$ produces electric fields of $-10^{10}$ V/m, if the bunch density is one-third as that of plasma (linear regime), and $-10^{11}$ V/m if the driving bunch density is $2.5n_0$ (blowout regime). We study the differences in the plasma wake created and what is an optimal position of the trailing electron bunch. Starting from initial 36 MeV trailing
bunch with \( n_b = 0.3n_0 \), its acceleration to 85 MeV is easily possible within 200 plasma periods. For greater times, the approximation for a driving electron bunch not to evolve in time becomes invalid and driving bunch distorts. Starting from initial 20 GeV trailing bunch with \( n_b = 0.3n_0 \), its acceleration to 21 GeV happens within 2000 plasma periods. When we increase the driving bunch density to \( n_b = 2.5n_0 \), starting from initial 20 GeV trailing bunch with \( n_b = n_0 \), its acceleration to 24 GeV occurs within 2000 plasma periods and plasma wake size is much larger, and therefore, the distance between driving and trailing bunches must be commensurately increased. Thus, it is possible to have the plasma wake-field acceleration in 1D and blowout regimes at the same time. It was established that, optimally, there should be approximately \((90–100)c/\omega_{pe}\) distance between the trailing and driving electron bunches, because in the 1D blowout regime, driving bunch’s wake is much longer than in 3D.

In summary, we show that in the linear regime and GeV energies, the accelerating electric field generated by the plasma wake is similar to the linear and MeV regimes. However, because the GeV energy driving bunch stays intact for a much longer time, the final acceleration energies are much larger in the GeV energies case. In the GeV energy range and the blowout regime, the wake’s accelerating electric field is much larger in amplitude compared with the linear case and also the plasma wake geometrical size is much larger. Therefore, the correct positioning of the trailing bunch is important to achieve the efficient acceleration of the trailing electron bunch.

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