Magnetic reconnection during collisionless, stressed, X-point collapse using particle-in-cell simulation

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Magnetic reconnection during collisionless, stressed, X-point collapse was studied using a kinetic, 2.5D, fully electromagnetic, relativistic particle-in-cell numerical code. Two cases of weakly and strongly stressed X-point collapse were considered. Here the descriptors “weakly” and “strongly” refer to 20% and 124% unidirectional spatial compression of the X-point, respectively. In the weakly stressed case, the reconnection rate, defined as the out-of-plane electric field in the X-point (the magnetic null) normalized by the product of external magnetic field and Alfvén speeds, peaks at 0.11, with its average over 1.25 Alfvén times being 0.04. During the peak of the reconnection, electron inflow into the current sheet is mostly concentrated along the separatrices until they deflect from the current sheet on the scale of electron skin depth, with the electron outflow speeds being of the order of the external Alfvén speed. Ion inflow starts to deflect from the current sheet on the ion skin depth scale with the outflow speeds about four times smaller than that of electrons. Electron energy distribution in the current sheet, at the high-energy end of the spectrum, shows a power-law distribution with the index varying in time, attaining a maximal value of −4.1 at the final simulation time step (1.25 Alfvén times). In the strongly stressed case, the magnetic reconnection peak occurs 3.8 times faster and is more efficient. The peak reconnection rate now attains the value 2.5, with the average reconnection rate over 1.25 Alfvén times being 0.5. Plasma inflow into the current sheet is perpendicular to it, with the electron outflow seeds reaching 1.4 Alfvén external Mach number and ions again being about four times slower than electrons. The power-law energy spectrum for the electrons in the current sheet attains now a steeper index of −5.5, a value close to those observed near the X-type region in the Earth’s magnetotail. Within about one Alfvén time, 2% and 20% of the initial magnetic energy is converted into heat and accelerated particle energy in the case of weak and strong stress, respectively. In both cases, during the peak of the reconnection, the quadruple out-of-plane magnetic field is generated, hinting possibly at the Hall regime of the reconnection. These results strongly suggest the importance of the collisionless, stressed X-point collapse as an efficient mechanism of converting magnetic energy into heat and superthermal particle energy. © 2007 American Institute of Physics. [DOI: 10.1063/1.2800854]

I. MOTIVATION OF THE STUDY

Magnetic reconnection is an important physical process which serves as one of the possible ways of converting energy stored in the magnetic field into heat and nonthermal, accelerated motion of plasma particles. This process operates in virtually all extragalactic, stellar, solar, space, and laboratory plasmas with varying degrees of importance. For example, in solar and stellar flares, magnetic reconnection plays a key role. In addition, it can be one of the main contributing factors to the solar coronal heating problem among other mechanisms such as wave dissipation. As far as plasma heating is concerned, in the collisional regime, be it Tokamak plasma or solar corona (which perhaps is better described as a collisionless medium), Spitzer resistivity is \( \propto T_e^{−3/2} \), where \( T_e \) is the electron temperature. Thus, in becoming hotter, plasma essentially starts to behave as a superconductor, i.e., further heating (increase in temperature) is impeded due to a decrease of the resistive properties. In this context, a direct conversion of magnetic energy into heat via reconnection seems rather attractive. As far as the particle acceleration is concerned, 50–80% of the energy released during solar flares is converted into the energy of accelerated particles. Thus, knowledge of the details of particle acceleration via magnetic reconnection (which is deemed a major mechanism operating solar and stellar flares) is also important. The main aspects of the reconnection process can be classified as resistive (collisional) or collisionless; spontaneous or forced; and/or steady or time-dependent. Each of these aspects have been studied extensively with varying levels of progress. For example, both steady and time-dependent resistive reconnection processes are very well studied (see, e.g., Ref. 1 and references therein), while it is only recently that some progress has been made in the study of collisionless reconnection (see, e.g., Refs. 2 and 3 and references therein). This can be explained by the fact that resistive reconnection is simpler in its nature. Changing the structure (connectivity) of magnetic field lines, which essentially is the reconnection, requires some mechanism (dissipation) to break the frozen-in condition. In the case of resistive reconnection, it is the \( \eta j^2 \) term. In fact, the rate at which reconnection proceeds is given by an inflow velocity into the dissipation region, \( v_m = (\delta/\Delta) v_A \), where \( \delta \) and \( \Delta \) are the width and length of the region, and \( v_A \) is the Alfvén speed. In the simplest resistive, steady model of Sweet-Parker,
$\delta = \eta^{1/2}$ and $\Delta \approx L$, where $L$ is the macroscopic system size. In the collisionless regime, however, other terms in Ohm’s law, such as the electron inertia term, the Hall term, and the electron pressure tensor become far more important than the usual $\eta^{-1}$ term upon which resistive reconnection models rely. Each of these terms has an associated spatial scale with them, i.e., a scale at which they start to play a dominant role. For example, the electron inertia term is associated with the electron skin depth $c/\omega_{pe}$, the Hall term is associated with the ion skin depth $c/\omega_{pi}$, while the electron pressure tensor is associated with the ion Larmor radius (see, e.g., p. 87 in Ref. 3 or p. 42 in Ref. 1).

One of the main outcomes of recent research in collisionless reconnection can be summarized by GEM$^{4,5}$ and Newton$^6$ reconnection challenges. These considered the same physical system: Harris-type equilibrium with antiparallel magnetic field, relevant to geomagnetic tail application, with finite initial magnetic perturbations in the case of the GEM challenge, and time-transient, inhomogeneous, driven inflow of magnetic flux in the case of the Newton challenge. The novelty of the approach was to use different numerical codes [magnetohydrodynamic (MHD), Hall MHD, hybrid, and particle-in-cell (PIC)] in order to pinpoint the essential physical mechanism that facilitates the reconnection. These works established that as long as dispersive whistler waves are included (these can only appear if one allows for the different dynamics for electrons and ions), the rate at which reconnection proceeds does not change, irrespective of which term breaks the frozen-in condition.

Yet another interesting analytical result corroborated by the numerical simulations is that in two-dimensional (2D) steady reconnection, the Petschek reconnection rate functional form remains the same when the Hall term is included (in the Hall MHD model), but what changes is the length from the X-point to the start of slow mode shocks.\(^7\) In MHD, this length is the half-width of the resistive region, $L_B$, but when the Hall term is included, the length is replaced by $L_R + c/\omega_{pi}$.\(^7\) This important result essentially provides an analytical scaling for the collisionless reconnection (when $\eta$ is zero) for such a physical system. Thus, future kinetic studies that use PIC simulation need to corroborate it. Reference 7 already tested this scaling law via MHD and Hall MHD simulation.

The importance of the electron inertia in the X-point with a finite out-of-plane guide magnetic field was investigated in Ref. 8. A particularly noteworthy result was that when the normalized collisionless electron skin depth $[c/\omega_{pe}L]$ exceeds the dimensionless resistive length scale ($S^{1/2}$), where $S$ is the Lindquist number, the energy in a shear Alfvén wave approaching an X-point is rapidly transformed into plasma kinetic energy and heat. Assuming solar coronal parameters, the normalized electron skin depth is $10^{-8}$ (assuming $L=10 \text{ Mm}$ and $c/\omega_{pe}=0.1 \text{ m}$), while the dimensionless resistive length scale is $3 \times 10^{-7}$ (in the corona typically $S=10^{13}$). Thus, the idea proposed in Ref. 8 may still be effective.

Previous works on particle acceleration focused mostly on test-particle-type calculations of the particle trajectories in different 2D$^{6,12}$ or more recently 3D$^{13-16}$ magnetic reconnection configurations. In such an approach, feedback on reconnection electromagnetic (EM) fields from the motion (spatial redistribution) of charged particles is ignored. Our approach does not suffer from this drawback, as we use a fully electromagnetic, relativistic PIC numerical code in which EM fields are calculated at each step from the spatial distribution of the charges.

According to Ref. 1, resistive time-dependent reconnection other than the well known tearing mode can be split into two main classes: X-type collapse, which was first considered in Ref. 17 (a decade before the tearing mode was discovered), and the Petschek-type theory developed in Ref. 18. Previous work on X-point collapse is well described in Chap. 7.1 of Ref. 1. Also see more recent work on the subject.\(^9\) A good example of the combination of analytical and numerical work on magnetic reconnection at stressed X-type neutral points can be found in Ref. 20. Boundary conditions used in Ref. 20 were such that they did not allow for flux or mass flow through the boundary. The studies with closed boundaries are physically justified by being isolated systems, whereas some of those with open boundaries lead to misleading results: for example, in an open system, a potential X-point can collapse due to inflow of energy from outside.

Initial analytical work on this topic considered unbounded self-similar solutions. These indicated that $E_0(0,0,t)$, the out-of-plane electric field at the magnetic null, which is the measure of reconnection rate, tends to infinity as $t \rightarrow \infty$ [e.g., Fig. (7.3) from Ref. 1]. The main outcomes of the previous, stressed X-point collapse in the resistive MHD (in the case of low-resistivity and low-beta) can be summarized as follows:

(i) The X-point collapse is different depending on whether initial stressing is weak or strong. In the weak case, the average reconnection rate scales as $1/\ln(\eta)$, where $\overline{\eta}=1/S$ is the dimensionless resistivity [$\eta=\eta/(V_{A0}L)=1/S$], while in the strong case it is independent of $\eta$.

(ii) There is an issue related to the efficiency of the process: Ref. 21 showed that for fast reconnection to occur, $\beta < \eta^{0.565}$, otherwise pressure in the current sheet chokes off the collapse process. For solar coronal conditions, $\eta_{\text{cor}} \approx 10^{-13}$, this requires $\beta < 10^{-8}$, which is too low to match the observed values circa 0.01–0.001. However, this shortcoming seems to be alleviated by the inclusion of nonlinear effects, i.e., the case of strong perturbations (strong stressing).\(^2\) If the compression is sufficiently large, magnetic pressure of the imploding wave expels trapped gas in the current sheet from its ends, allowing it to thin and faster reconnection to occur.

The motivation of the present study is fivefold:

(i) Naturally, different boundary and initial conditions produce different scaling of the reconnection rate, e.g., with resistivity (in the case of resistive reconnection). To the best of our knowledge, X-type collapse by a uniform stress (compression) along one of the axes, as described in Chap. 2.1 in Ref. 1, has not been investigated numerically either in the case of resistive (MHD) reconnection or in the collisionless regime.

(ii) This type of stress (compression) is likely to occur in the framework of a flare model (see details below).
(iii) To the best of our knowledge, most of the previous collisionless reconnection studies considered an antiparallel, Harris-type magnetic field configuration, which is more relevant to Earth magnetotail application. For solar and stellar flares, the X-type configuration is more relevant. For solar and stellar flares, the X-type configuration is more relevant.

(iv) Some of the models of coronal mass ejections \(^{22}\) that use the motion of photospheric footpoints as a driver end up with a situation physically similar to stressed X-point collapse (strictly speaking, Y-points occur there).

(v) We aimed to investigate the properties of the accelerated particles in the current sheet using self-consistent electromagnetic fields, as a further extension of the test-particle-type approach.

II. SIMULATION MODEL

A. Stressed X-point reconnection model

Figure 1 represents magnetic field line geometry considered in this paper. In order to study stressed magnetic reconnection, we consider a magnetic X-point collapse that may naturally occur during solar flares [see the sketch in Fig. 1(a)]. Such a stressed X-point may occur, e.g., if photospheric footpoints of the coronal loops move toward each other (e.g., pushed by convective motions) or some compression from the sides in the corona forces them to do so. We study the dynamics of such a stressed X-point by means of a kinetic, 2.5D, fully electromagnetic, relativistic particle-in-cell numerical code. We focus attention on the local region inside the dashed box which is the region that our study tries to mimic by uniform stress in one direction [see Fig. 1(c)]. If there is no stress from the sides, the considered magnetic configuration is stable [Fig. 1(b)]. Interestingly, Ref. 23 investigated what happens when a fast magnetosonic shock wave associated with a coronal mass ejection collides obliquely with a coronal streamer with a stable current sheet. Their setup is somewhat analogous to ours, but much more violent, as we only consider X-point compression, while they blasted it with an Alfvén Mach 6 shock.

The initial magnetic field configurations used are

\[
B_x = \frac{B_0}{L} y, \quad B_y = \frac{B_0}{L} \alpha^2 x, \quad B_z = 0, \quad (1)
\]

where \(B_0\) is the magnetic field intensity at the distance \(L\) from the X-point for \(\alpha = 1.0\), \(L\) is the global external length-scale of reconnection, and \(\alpha\) is the stress parameter (see Chap. 2.1 in Ref. 1). In addition, the uniform current is imposed at \(t=0\) in the \(z\) direction,

\[
j_z = \frac{B_0}{\mu_0 L} (\alpha^2 - 1). \quad (2)
\]

This current appears because of the stress imposed on magnetic field lines.

B. PIC simulation code

The simulation code used here is the 2.5D relativistic and fully electromagnetic PIC code, modified from the 3D TRISTAN code.\(^{24}\) In this code, both the electron and ion dynamics are described as particles. The equation of motion for each particle is solved using self-consistent electromagnetic fields,

\[
\frac{d\vec{v}_{si}}{dt} = \frac{q_s}{m_i}(\vec{E} + \vec{v}_{si} \times \vec{B}), \quad (3)
\]

\[
\frac{d\vec{r}_{si}}{dt} = \vec{v}_{si}, \quad (4)
\]

\[
\frac{\partial \vec{E}}{\partial t} = c^2 \nabla \times \vec{B} - \frac{1}{\epsilon_0} \sum_i \vec{j}_i, \quad (5)
\]

\[
\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}, \quad (6)
\]

where \(\vec{E}, \vec{B}, \vec{j}_i, \vec{v}_{si},\) and \(\vec{r}_{si}\) are electric and magnetic fields, current density, particle velocity, and position, respectively. The subscript \(s\) represents species of plasma, that is, \(s = e\) for electron and \(s = i\) for ion. The subscript \(i\) indicates the \(i\)th particle index. The other quantities, \(q_s, m_i, c,\) and \(\epsilon_0\), are the charge and mass of a plasma particle, the speed of light, and the vacuum permittivity, respectively. In addition to Eqs. (3)–(6), \(\nabla \cdot \vec{B} = 0\) and \(\nabla \cdot \vec{E} = \rho_s/\epsilon_0\) must be satisfied initially (\(\rho_s\) is charge density, which is taken as 0). The latter two conditions are automatically satisfied at all times due to the nature of the numerical scheme used.\(^{24}\) Electromagnetic fields that satisfy Eqs. (5) and (6) update particle velocity through the equation of motion (3), which in turn updates particle positions (4). Repeating this procedure many times mimics plasma particle dynamics in self-consistent electromagnetic fields.

We started by testing our code, reproducing published GEM challenge results.\(^{4,5}\) Figure 2 shows the time evolution of the reconnected magnetic flux difference between O and X-lines. Open squares correspond to the published GEM challenge values. We gather from this graph that the match between our simulation results and that of GEM challenge\(^{4,5}\) is...
The length of the system in two dimensions is \( L_x = L_y = 400 \Delta \) (this is excluding so called ghost cells needed for updating the boundary conditions), where \( \Delta = 1 \) is the simulation grid size corresponding to electron Debye length, \( \lambda_p = v_{pe}/\omega_{pe} = 1 \Delta \) \((v_{pe} \text{ is electron thermal velocity and } \omega_{pe} \text{ is electron plasma frequency})\). The global external length scale of reconnection is set to \( L = 200 \Delta \). The number density is fixed at \( n_0 = 100 \) electron-ion pairs per cell. Both electrons and ions are uniformly distributed throughout the system, hence the total number is 1.6 million pairs.

Zero-gradient boundary conditions are imposed on both the electric and magnetic fields in the \( x \) and \( y \) directions. Also, the tangential component of electric field was forced to zero, while the normal component of the magnetic field was kept constant, both at the boundary. This ensures that there is no magnetic flux through the simulation boundary, i.e., the system is isolated and \(-f_x = f_y = 0 \) \((f_x = \text{electron } \text{c}) \text{lectron}\) \text{x}\text{ electric field} \text{flux} \text{through}\text{ the}\text{ boundary}\text{.} \text{The}\text{ latter}\text{ sum}\text{ is}\text{ the}\text{ magnetic}\text{ flux}\text{ on}\text{ the}\text{ boundary.}\text{ Reflecting}\text{ boundary}\text{ conditions}\text{ are}\text{ imposed}\text{ on}\text{ particles}\text{ in}\text{ both}\text{ the}\text{ }x\text{ and }y\text{ directions.}\text{ The}\text{ latter}\text{ ensures}\text{ that}\text{ there}\text{ is}\text{ no}\text{ mass}\text{ flow}\text{ across}\text{ the}\text{ boundary.}\text{ The}\text{ simulation}\text{ time}\text{ step} \text{is} \omega_{pe} \Delta t = 0.05.\text{ The}\text{ ion-to-electron}\text{ mass}\text{ ratio}\text{ is} \text{ } m_i/m_e = 100.\text{ The}\text{ electron}\text{ thermal velocity}\text{ to}\text{ speed}\text{ of}\text{ light}\text{ ratio} \text{is} v_{te}/c = 0.1.\text{ The}\text{ electron}\text{ and}\text{ ion}\text{ skin}\text{ depths}\text{ are} c/\omega_{pe} = 10 \Delta\text{ and} c/\omega_{pi} = 100 \Delta,\text{ respectively.}\text{ The}\text{ electron}\text{ cyclotron}\text{ frequency}\text{ to}\text{ plasma}\text{ frequency}\text{ ratio} \text{is} \omega_{ce}/\omega_{pe} = 1.0\text{ for magnetic field intensity,} \text{ } B = B_0.\text{ The}\text{ latter}\text{ ratio}\text{ is}\text{ indeed}\text{ close}\text{ to} \text{unity in the solar corona, while it is much bigger than unity in the Earth’s magnetosphere.} \text{Electron}\text{ and}\text{ ion}\text{ Larmor radii}\text{ are} v_{te}/\omega_{ce} = 1 \Delta\text{ and} v_{ti}/\omega_{ci} = 10 \Delta,\text{ where} v_{ti}\text{ is the thermal velocity}\text{ of}\text{ ions}\text{ and}\text{ electrons}\text{ are}\text{ the}\text{ same,} \text{ } T_e = T_i.\text{ At the boundary,} the\text{ plasma} \beta = 0.02\text{ and the Alfvén velocity} V_{A0}/c = 0.1.\text{ Naturally these vary across the simulation box as the background magnetic field is a function of} x\text{ and} y.\text{ In what follows, for visualization purposes, all spatial coordinates will be normalized by electron skin depth c/\omega_{pe}, while time is normalized by the inverse of plasma electron frequency } \omega_{pe}^{-1}.\text{ }

III. SIMULATION RESULTS

We investigated three X-point collapse cases with different stress parameters: \( \alpha = 1.00 \) (stable), 1.20 (weakly stressed), and 2.24 (strongly stressed). For \( \alpha = 1.00 \), we confirmed that the system is stable at least for \( t \leq 500 \), and no magnetic reconnection takes place. Reference 21 has shown that when perturbations (exerted stress on an X-point) are small \( e = (1 - \alpha^2) < \bar{\eta} \), then the (average) reconnection rate scales as \( 1/\ln(\bar{\eta}) \), while if they are large \( e = (1 - \alpha^2) \geq \bar{\eta} \), then the reconnection rate is independent of \( \bar{\eta} \). This difference in behavior points to a different relative importance of the physical mechanisms in action. The two different (weakly and strongly compressed) cases studied in this paper attempt the same approach as in Ref. 21. However, one should realize that we use a vastly different physical description. Reference 21 uses resistive MHD, while our PIC (fully kinetic) approach is collisionless, and, in turn, in our case \( \bar{\eta} = 0 \). However, it has been observed before that often scattering of plasma particles from the magnetic field lines plays an effective role of collisions. For example, Refs. 25 and 26 have shown that the Alfvén wave dissipation in the collisionless case (using the same PIC, kinetic code) follows a scaling law that is the same as in the resistive MHD case.\(^{27}\) At the same time, this is not to say that we are confident that the effective particle scattering off the magnetic fields, which mimics resistive (collisional) effects, is the mechanism that breaks down the frozen-in condition here. The issue of which term in the generalized Ohm’s law is responsible for the reconnection in the collisionless stressed X-point collapse will be studied separately.

A. Weakly stressed X-point case (\( \alpha = 1.20 \))

It should be noted that \( \alpha \), the measure of stress, is essentially the aspect ratio of the compression, i.e., since the limiting magnetic field lines are given by \( y = ax \), \( \alpha = 1.2 \) means 1.2–1.0 = 0.2, i.e., 20\% compression of the X-point in \( x \) direction. Similarly, the \( \alpha = 2.24 \) case represents 124\% compression, i.e., 1.24 times stressed.

Also, because a change in \( \alpha \) means a change in the strength of the magnetic field on the boundary, the external Alfvén speed also changes. For example, we fixed the Alfvén velocity \( V_{A0}/c = 0.1 \) for \( \alpha = 1 \), but for \( \alpha > 1 \), \( V_{A0}/c = \sqrt{1 + \alpha^2}/2 \times 0.1 \). Thus for \( \alpha = 1.20 \), \( V_{A0}/c = 0.124 \), but for \( \alpha = 2.24 \), \( V_{A0}/c = 0.36 \).

Figure 3(a) shows the time evolution of the reconnection rate, defined as the out-of-plane electric field in the X-point, \( E_z(0, 0, t) \) normalized by the product of the external magnetic field and Alfvén speeds (both at the boundary), \( E_0,V_{A0}/c \), for \( \alpha = 1.20 \) (solid line) and \( \alpha = 1.00 \) (dotted line). Because usually PIC simulation suffers for large (ther-
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X-point collapse or reconnection occurs outside the X-point to the outer boundary of the simulation, while the case of current prescribed by Eq. 1 remains current-free throughout the initial value, $j_0 = 1.0$, a relatively small value. The total current density is normalized by the initial value, $j_0 = n_0 v d_0$. We gather from this figure that for the case of $\alpha = 1$, no $E_x$ is generated, and as such the configuration is stable and no X-point collapse or reconnection occurs (dotted line). On the contrary, for the case of $\alpha = 1.2$ we see that the reconnection rate peaks at about 0.11 at $t = 170$, which is the same as $t/\tau_\alpha = 0.85$. Here $\tau_\alpha$ is the Alfven time—the distance from the outer boundary to the X-point (20) divided by the Alfven speed at the boundary ($V_{\text{A}0} = 0.1 c$). We also calculated the average reconnection rate based on Fig. 3(a). It is defined as

$$E_{av} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \frac{E_x(0,0,t)}{E_0} \, dt,$$

where $t_f = 250$ is the final simulation time. This yields $E_{av} = 0.04$, a relatively small value.

Figure 3(b) shows the time evolution of total current density, $j_z$, in the X-point for $\alpha = 1.20$ (solid line) and $\alpha = 1.0$ (dotted line). The total current density is normalized by the initial value, $j_0 = n_0 v d_0$, $v_{d0}$ is the drift velocity. As expected, the $\alpha = 1$ case remains current-free throughout the simulation, while $\alpha = 1.2$ produces a peak current of $j_z/j_0 = 15$ and then subsequently decays off.

It should be noted that we have performed one additional numerical run with $\alpha = 1.2$, but without imposing initial $j_0$ current prescribed by Eq. (2). This is because, e.g., one might expect that in an X-point above the arcade of loops in the solar corona, only compression (stress) of the magnetic field from the two sides is likely. Such perturbation violates the equations at $t = 0$. This results in a large spike of magnitude 0.22 at $t = 5$ in the equivalent version of Fig. 3(a) (not presented here). Otherwise, for $t > 5$ there was no noticeable difference between the cases with and without imposing the initial current. Thus, in what follows we only discuss cases with the current imposed at $t = 0$.

Figure 4 shows the time evolution of the spatial distribution of total current density, $j_z$, in the $x$-$y$ plane at (a) $t = 0$, (b) 100, (c) 170, and (d) 250 for $\alpha = 1.20$. The total current density is normalized by the initial value, $j_0 = n_0 v d_0$. The current peaks in the current sheet at $t = 170$, as seen in Fig. 4(c). In the later phases, the current sheet tends to decay [Fig. 4(d)]. This figure is useful for visualizing the spatial dimensions of the current sheet. In turn, this enables us to make the following useful estimate: From Fig. 4(c) (peak current time snapshot), we gather that the width of the current sheet (in horizontal direction) is about the electron skin depth $\delta = 1.4$, while its length (in the vertical direction) is about $\Delta = 10$. To be precise, we actually measured $\delta$ and $\Delta$ by more accurate means: we looked at line plots of $j_z(x,0)$ and $j_z(0,y)$, respectively, at $t = 170$, and measured the appropriate half-width of the $j_z(x,0)$ and $j_z(0,y)$ peak (in both the $x$ and $y$ directions). The ratio of the two gives the inflow Alfven Mach number $M_A = \delta/\Delta = 0.14$. This is very close to the peak value of $E_x/E_0 = v/V_{\text{A}0} = M_A = 0.11$ [from Fig. 3(a)]. We thus measured the peak reconnection rate by two independent means directly from the simulation [using Fig. 3(a)] and using the steady reconnection formula $M_A = \delta/\Delta$ [e.g., Eq. (3.1) from Ref. 3].

FIG. 3. (a) Time evolution of the reconnection rate, defined as the out-of-plane electric field in the X-point, $E_x(0,0,t)$, normalized by the product of external magnetic field and Alfven speeds (both at the boundary for $\alpha = 1.00$) for $\alpha = 1.20$ (solid line) and $\alpha = 1.00$ (dotted line). (b) Time evolution of total current density, $j_z$, in the X-point for $\alpha = 1.20$. Note that the final simulation time $t = 250$ corresponds to $t/\tau_\alpha = 1.25$, with the latter being the Alfven time.

FIG. 4. Time evolution of the spatial distribution of total current density, $j_z$, in the X-Y plane at (a) $t = 0$, (b) 100, (c) 170, and (d) 250 for $\alpha = 1.20$. The total current density is normalized by the initial value, $j_0 = n_0 v d_0$. The current peaks in the current sheet at $t = 170$, as seen in Fig. 4(c). In the later phases, the current sheet tends to decay [Fig. 4(d)]. This figure is useful for visualizing the spatial dimensions of the current sheet. In turn, this enables us to make the following useful estimate: From Fig. 4(c) (peak current time snapshot), we gather that the width of the current sheet (in horizontal direction) is about the electron skin depth $\delta = 1.4$, while its length (in the vertical direction) is about $\Delta = 10$. To be precise, we actually measured $\delta$ and $\Delta$ by more accurate means: we looked at line plots of $j_z(x,0)$ and $j_z(0,y)$, respectively, at $t = 170$, and measured the appropriate half-width of the $j_z(x,0)$ and $j_z(0,y)$ peak (in both the $x$ and $y$ directions). The ratio of the two gives the inflow Alfven Mach number $M_A = \delta/\Delta = 0.14$. This is very close to the peak value of $E_x/E_0 = v/V_{\text{A}0} = M_A = 0.11$ [from Fig. 3(a)]. We thus measured the peak reconnection rate by two independent means directly from the simulation [using Fig. 3(a)] and using the steady reconnection formula $M_A = \delta/\Delta$ [e.g., Eq. (3.1) from Ref. 3].
The close match points to the fact that a nearly steady reconnection regime is achieved in the vicinity of the peak at $t=170$.

Figure 5 shows the time evolution of the spatial distribution of the out-of-plane magnetic field, $B_z$, at (a) $t=0$, (b) 100, (c) 170, and (d) 250 for $\alpha=1.20$. The magnetic field intensity is normalized by the initial value, $B_0$.

In Fig. 6, we show the dynamics of individual magnetic field lines. We tried to trace the dynamics of several magnetic field lines in order to visualize the reconnection process. We can clearly see that magnetic field lines come toward each other in the $x$ direction, reconnect, and move apart in the $y$ direction.

Figure 7 shows electron (a) and ion (b) flows at the peak time of the reconnection. It is rather instructive to see that this figure in effect corroborates the sketch from Ref. 3 [see their Fig. (3.1)]. In particular, it shows that the electron and ion flow are clearly separated on the two different spatial scales—electron skin-depth and ion skin-depth; note that since here the mass ratio is 100, $c/\omega_{pi}=10c/\omega_{pe}$. The noticeable difference is caused because they considered an initial background magnetic field of Harris-type (antiparallel), while we study the X-point. Naturally, in our case electron inflow into the current sheet is mostly concentrated along the separatrices until they deflect from the current sheet on the scale of electron skin depth, with the electron outflow speeds being of the order of the external Alfvén speed $0.13c$. Ion inflow starts to deflect from the current sheet on the ion skin depth scale with the outflow speeds about four times smaller ($0.03c$) than that of electrons. As argued by Ref. 29, it is the difference in these two flows that generates the observed quadruple out-of-plane magnetic field.

Figure 8 shows the local electron energy distribution function near the current sheet at $t=0$ (dashed curve) and $t=250$ (solid curve) for $\alpha=1.20$. We performed a numerical fit to the high-energy part of the distribution function ($E>0.08m_{\text{e}}c^2=41$ keV) and found that the electron distribution has a power-law form, i.e., in particular the best fit is provided by $f=dN/dE\propto E^{-4.1}$ (straight solid line). In general, the direct observational evidence for the relation between reconnection and particle acceleration is hard to obtain because the acceleration length scale is of the order of ion skin-depth. However, recently it became possible to directly measure the electron energy spectrum in the vicinity of the X-type region in the Earth’s magnetotail. It was found that from about 2 keV to about 200 keV, the power-law index was between $-4.8$ and $-5.3$ (changing in the course of time of the observation). It is interesting to note that both our power-law index ($-4.1$) and the energy ($0.2 \times m_{\text{e}}c^2\approx 0.2 \times 511\approx 100$ keV) at the high end of the spectrum are close to the observed values in Ref. 30. Note that since the parameters of our simulation are for the solar corona ($\omega_{ci}/\omega_{pe}=1.0$), and are not even totally realistic for that (e.g., to sim-
plify numerical simulation, our \( m_e/m_p, v_e/c, \) and \( V_{A0}/c \) ratios are not entirely realistic), the absolute values of our simulation quoted in keV should be taken with caution (when comparing them to Earth’s magnetotail results). Yet we are confident in the correctness of the obtained power-law indexes. Also noteworthy is the fact that the whistler wave turbulence, based on the Fokker-Planck equation for the electron distribution function, subject to a zero-flux boundary condition \( \) same as ours by chance \( \), produces\(^3\) a similar power-law energy spectrum. Recall that it is standing whistler waves that are thought to mediate reconnection in the Hall regime.

**B. Strongly stressed case \( \alpha=2.24 \)**

In this subsection, we present the results of numerical simulation for the case of \( \alpha=2.24 \), which is regarded as a case of a strongly stressed X-point.

Figure 9 shows the numerical simulation results as in Fig. 3 but for \( \alpha=2.24 \). To avoid repetition, we omitted the \( \alpha=1 \) case (dotted line in Fig. 3). We gather from panel (a) that now the reconnection rate attains a value of 2.5 at \( t=45 \, (0.225 \tau_A) \). In the strongly stressed case, in contrast to the weakly stressed one, \( E_r \) rebounds and an oscillation is established. We calculated the average reconnection rate based on Fig. 9 using the definition from Eq. (7). The result is \( E_{av}=0.5 \), a value \( \approx 10 \) times bigger than in the weakly stressed case, indicative of more efficient reconnection. Also, much stronger current density, \( j_z/j_0=55 \), is generated compared to the weak case panel (b), and the reconnection peak occurs \( 170/45=3.8 \) times earlier. Similar oscillations of current and electric field were used as a mechanism for interpreting the peculiar hard-x-ray \( (>25 \text{ keV}) \) solar flare, which is believed to be produced by a nonthermal electron beam.\(^{32,33}\)

Figures 10(a) and 10(b) show the spatial distribution of the \( j_z \) current at the peak of the reconnection \( t=45 \), and after the current sheet decayed, \( t=70 \) (the first bounce; see Fig. 9). The noticeable difference from panels (c) and (d) in Fig. 4 is that now the current sheet becomes longer and thinner \( \) the same conclusion was reached in Ref. 21, in that large perturbations \( \) strong compression \) yield current sheet thinning and the onset of more efficient reconnection. As in the weakly stressed case, we also did the following calculation: Using

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data from Fig. 10(a) (peak current time snapshot), the width and length of the current sheet were estimated by looking at the half-width of the \( j_z(x,0) \) and \( j_z(0,y) \) peak (in both the \( x \) and \( y \) directions). The result is width \( \delta = 0.4 \) and length \( \Delta \approx 17 \). The ratio of the two gives the inflow Mach number \( M_A = \delta / \Delta = 0.02 \). This is 125 times smaller than the peak reconnection rate of 2.5 from Fig. 9(a) and 25 time smaller than the average reconnection rate of 0.5 from the same figure. This discrepancy can only be attributed to the fact that the formula \( M = \delta / \Delta \) only applies to the steady reconnection process. In the case of \( \alpha = 1.2 \), the reconnection process was relatively steady, thus the latter formula proved a good match, but for \( \alpha = 2.24 \) the process is too dynamic and thus \( M_A = \delta / \Delta \) as a measure of reconnection fails. Figures 10(c) and 10(d) again show a familiar quadruple out-of-plane magnetic field structure, but now it is much more elongated. Panels (e) and (f), in which we trace individual magnetic field lines at two different times, provide additional proof that the reconnection takes places and that the current sheet is now much longer and thinner.

Figure 11 is an analog of Fig. 7 but for \( \alpha = 2.24 \). A noteworthy difference from the weak case is that the plasma inflow into the current sheet is perpendicular to it, with the electron outflow speeds reaching an external Alfvén Mach number of \( 0.5 / 0.36 = 1.4 \) [note the arrow length in Fig. 11(a)], and the ions again being about four times slower than the electrons. Note that now \( V_{A0}/c = 0.36 \).

Figure 12 shows the local electron energy distribution function in the current sheet at \( t=0 \) (dashed curve) and \( t = 250 \) (solid curve) for \( \alpha = 2.24 \). Note that the dashed curves (corresponding to \( t=0 \)) in Figs. 8 and 12 are different. This is due to the fact that different \( \alpha \)'s mean different initial \( j_0 \) current [see Eq. (2)], which contribute to the calculation of the distribution function. We performed a numerical fit to the high-energy part of the distribution function \( (E > 2.4m_ec^2 = 1.226 \text{ MeV}) \) and found that the electron distribution has a power-law index of \(-5.5\), which is quite close to the observed power-law range of \(-4.8\) and \(-5.3\). Note that now the attained energies are much higher, \( 4 \times m_ec^2 = 0.2 \times 511 \text{ keV} = 2 \text{ MeV} \) (at the high end of the spectrum). Again, we note that values quoted in keV and MeV should be taken with caution (see the discussion of Fig. 8 above).
We studied magnetic reconnection during collisionless, stressed, X-point collapse using a kinetic, 2.5D, fully electromagnetic, relativistic particle-in-cell numerical code. We investigated two cases of weakly and strongly stressed X-points. Here by weakly and strongly we mean 20% and 124% unidirectional spatial compression of the X-point, respectively. In the weakly stressed case, the reconnection rate, defined as the out-of-plane electric field in the magnetic null normalized by the product of external magnetic field and Alfvén speeds, peaks at 0.11 (at 0.85 Alfvén times), with its average over 1.25 Alfvén times being 0.04. We found that during the peak of the reconnection, electron inflow into the current sheet is mostly concentrated along the separatrices until they deflect from the current sheet on the scale of the electron skin depth, with the electron outflow speeds being of the order of the external Alfvén speed. Ion inflow starts to deflect from the current sheet on the ion skin depth scale with the outflow speeds about four times smaller than that of electrons. Electron energy distribution in the current sheet, at the high-energy end of the spectrum, shows a power-law distribution with an index that varies in time, attaining a maximal value of −4.1 at the final simulation time step (this corresponds to 1.25 Alfvén times).

The obtained results in the strongly stressed case show that the magnetic reconnection peak occurs about 3.8 times faster (at 0.225 Alfvén times) and is more efficient. The peak reconnection rate now attains the value 2.5 (at 0.225 Alfvén times), with the average reconnection rate over 1.25 Alfvén times being 0.5. Plasma inflow into the current sheet is perpendicular to it, with the electron outflow seeds reaching 1.4 Alfvén external Mach number and ions again being about four times slower than electrons. The power-law energy spectrum for the electrons in the current sheet attains now a steeper index of −5.5. This is close to the typical observed power-law indexes in the vicinity of the X-type region in the Earth’s magnetotail.

The reconnection rate versus time figures in both cases indicates that the reconnection has bursty, time-transient behavior. In the two cases considered, 2% (in weakly stressed) and 20% (in strongly stressed) of the initial magnetic energy is converted into heat and energy of accelerated particles, respectively, both within about one Alfvén time. This is somewhat similar to the previous resistive MHD analog of the present study, in that small perturbation (weak stress) initiates X-point collapse, but then the reconnection process is choked off by the gas pressure. At present, it is unclear, however, whether we seem to observe similar behavior in our simulation. After all, e.g., Petschek mechanism reconnection can be choked off by a different (other than pressure) mechanism (when the diffusion-region inflow magnetic field gets too small).

We also found that in both cases, during the peak of the reconnection, the quadruple out-of-plane magnetic field is generated. This most likely suggests that the Hall regime of the reconnection takes place.

In addition to the fundamental interest of converting magnetic energy into heat and the energy of accelerated par-
particles, these results are significant for, e.g., the solar coronal heating problem. It has been estimated that even if only 2% of the magnetic energy in the solar corona is converted into heat, the coronal heating problem would be solved. Also, it is known that the typical resistive diffusion time in the corona is $10^{15}$ Alfvén times ($10^8$ years), while the time-transient phenomena such as flares (one of the possible candidates of coronal heating) occur on time scales of 10–100 Alfvén times. Our results for the strongly stressed case suggest that 20% of initial magnetic energy can be released in just 1 Alfvén time. On the one hand, one has to realize that the obtained results are for an X-point configuration, and naturally the bulk of the solar corona is not made of solely X-points. On the other hand, it is also known that heating of the active regions would provide about 82% of the coronal heating budget.\(^3\)

In turn, given the fact that X-points are a common occurrence in the active regions, our results seem to be of importance for solving the coronal heating problem.

It is important to stress that the main result of the 20% conversion of the initial magnetic energy into heat and energy of superthermal particles, within about one Alfvén time, is obtained in the collisionless regime, thus this result does not suffer from any uncertainty in the anomalous resistivity, as in the case of resistive MHD reconnection.

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